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THE TEMPERATURE FIELD AND THE THERMOELASTIC STATE IN A PLATE CONTAINING A THIN-WALLED ELASTIC INCLUSION*

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A plane problem of heat conductivity and thermoelasticity is considered for a plate containing a rectilinear, thin-walled elastic inclusion of finite length. The problem is reduced to a system of two singular, integrodifferential Prandtl-type equations, which are solved using the method of orthogonal polynomials. A numerical analysis of the solution is given.

1. Formulation of the problem. We consider an isotropic plate containing a foreign, thin-walled rectilinear inclusion of length $2a$, thickness $2h$, acted upon by the thermal parameters only (heat flux at infinity, concentrated heat sources). It is assumed that the side surfaces of the plate are thermally insulated, and that perfect thermal contact and force coupling exist between the edges of the inclusion and the surrounding material. We require to determine and study the effect of the inclusion on the magnitude and character of the distribution of the temperature field and thermoelastic state in the plate.

To solve the problem, we shall employ a Cartesian xOy-coordinate system the axes of which are directed along the axes of the inclusion (Fig.1). We denote the length of the inclusion by L, and the quantities referring to the inclusion by a subscript zero. The plus and minus indices denote the boundary values of the functions at the upper and lower edges of the inclusion, respectively.

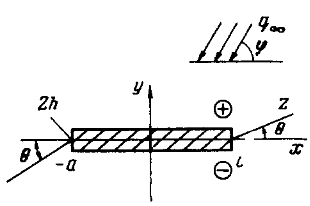

Fig.1

The conditions of the force coupling and thermal contact between the inclusion and the surrounding material have the form

$$(\sigma_y - i\tau_{xy})_0^{\pm} = (\sigma_y - i\tau_{xy})^{\pm}, \quad (u+iv)_0^{\pm} = (u+iv)^{\pm} \quad \text{on } L \tag{1.1}$$

$$(T+i\eta)_0^{\pm} = (T+i\eta)^{\pm}, \quad k_0 \frac{\partial}{\partial y}(T+i\eta)_0^{\pm} = k\frac{\partial}{\partial y}(T+i\eta)^{\pm} \quad \text{on } L \tag{1.2}$$

where η is an auxilliary harmonic function /1/ and k_0, k are the heat conductivity coefficients of the inclusion and the plate materials, respectively.

2. Problem of heat conduction. According to /1/, the temperature field in a homogeneous isotropic plate can be found using the formula

$$F_1(z) + Q_1(\bar{z}) = T + i\eta, \quad F(z) + Q(\bar{z}) = \frac{\partial}{\partial x}(T+i\eta), \quad F(z) - Q(\bar{z}) = -i\frac{\partial}{\partial y}(T+i\eta) \tag{2.1}$$

$$(F(z) = F_1'(z), \; Q(z) = Q_1'(z))$$

where $F_1(z)$ and $Q_1(z)$ are piecewise holomorphic functions. Since we consider a thin-walled inclusion, we can neglect the quantities which are very small compared with h, and use (2.1) to write

$$\frac{\partial}{\partial x}(T+i\eta)_0^+ + \frac{\partial}{\partial x}(T+i\eta)_0^- = 2g(x), \quad x\in L, \quad \frac{\partial}{\partial x}(T+i\eta)_0^+ - \frac{\partial}{\partial x}(T+i\eta)_0^- = 2h\rho'(x), \quad x\in L \tag{2.2}$$

$$\frac{\partial}{\partial y}(T+i\eta)_0^+ + \frac{\partial}{\partial y}(T+i\eta)_0^- = 2\rho(x), \quad x\in L, \quad \frac{\partial}{\partial y}(T+i\eta)_0^+ - \frac{\partial}{\partial y}(T+i\eta)_0^- = -2hg'(x), \quad x\in L$$

where $g(x)$ and $\rho(x)$ are functions to be determined.

Satisfying the conditions (1.2) with help of the relations (2.1) and taking (2.2) into account, we obtain the following boundary value problems for determining the piecewise holomorphic functions $F(z)$ and $Q(z)$ with the line of discontinuity L:

$$[F(x)+Q(x)]^+ + [F(x)+Q(x)]^- = 2g(x), \; x\in L, \quad [F(x)-Q(x)]^+ + [F(x)-Q(x)]^- = -2i\frac{k_0}{k}\rho(x), \; x\in L \tag{2.3}$$

$$[F(x)+Q(x)]^+ - [F(x)+Q(x)]^- = 2ih\frac{k_0}{k}[g'(x) - g_1(x)], \quad x\in L \tag{2.4}$$

$$[F(x)-Q(x)]^+ - [F(x)-Q(x)]^- = 2h[\rho'(x) - \rho_1(x)], \quad x\in L$$

*Prikl.Matem.Mekhan.,44,No.2,338-345,1980 where

$$g_1(x)=[F_2'(x)+Q_2'(x)]\,\varepsilon_1,\quad \varepsilon_1=\frac{\min(k_0,k)}{k_0},\quad \rho_1(x)=[F_2'(x)-Q_2'(x)]\,i\varepsilon_2,\quad \varepsilon_2=\frac{\min(k_0,k)}{k}$$

$F_2(x)$ and $Q_2(x)$ are known functions which yield a solution to the problem of heat conduction for the same plate without inclusion.

Solving the problem of linear coupling (2.4), we find

$$F(z)=\frac{h}{2\pi}\left[\frac{k_0}{k}\int_{-a}^{a}\frac{[g'(x)-g_1(x)]\,dx}{x-z}-i\int_{-a}^{a}\frac{[\rho'(x)-\rho_1(x)]\,dx}{x-z}\right]+F_2(z) \tag{2.5}$$

$$Q(z)=\frac{h}{2\pi}\left[\frac{k_0}{k}\int_{-a}^{a}\frac{[g'(x)-g_1(x)]\,dx}{x-z}+i\int_{-a}^{a}\frac{[\rho'(x)-\rho_1(x)]\,dx}{x-z}\right]+Q_2(z)$$

Substituting the expressions for the functions $F(z)$ and $Q(z)$ given in (2.5) into (2.3), we obtain two singular, integrodifferential Prandtl-type equations for determining the unknown functions $g(x)$ and $\rho(x)$

$$g(x)-\frac{hk_0}{\pi k}\int_{-a}^{a}\frac{[g'(t)-g_1(t)]\,dt}{t-x}=F_2(x)+Q_2(x),\quad x\in L \tag{2.6}$$

$$\frac{k_0}{k}\rho(x)-\frac{h}{\pi}\int_{-a}^{a}\frac{[\rho'(t)-\rho_1(t)]\,dt}{t-x}=i\,[F_2(x)-Q_2(x)],\quad x\in L$$

We seek the solution of (2.6) in the form

$$g(ax)=g_2(ax)-\sqrt{1-x^2}\sum_{m=1}^{\infty}\frac{1}{m}X_mU_{m-1}(x),\quad \rho(ax)=\rho_2(ax)-\sqrt{1-x^2}\sum_{m=1}^{\infty}\frac{1}{m}Y_mU_{m-1}(x),\quad |x|\leqslant 1 \tag{2.7}$$

$$g_2(x)=[F_2(x)+Q_2(x)]\,\varepsilon_1,\quad \rho_2(x)=[F_2(x)-Q_2(x)]\;i\varepsilon_2$$

where X_m and Y_m are unknown coefficients and $U_m(x)$ are Chebyshev polynomials of second kind. From (2.6) and (2.7) we follow /2/ to arrive at two infinite, quasiregular systems of linear algebraic equations for determining the coefficients X_m and Y_m of the expansions

$$\sum_{m=1}^{\infty}R(m,n)\,X_m+\frac{\pi hk_0}{2ak}X_n=(\varepsilon_1-1)\,D_n^+,\quad \frac{k_0}{k}\sum_{m=1}^{\infty}R(m,n)\,Y_m+\frac{\pi h}{2a}Y_n=i\,(\varepsilon_2-1)\,D_n^-\quad (n=1,2,\ldots) \tag{2.8}$$

$$D_n^{\pm}=\int_{-1}^{1}[F_2(ax)\pm Q_2(ax)]\sqrt{1-x^2}\,U_{n-1}(x)\,dx$$

$$R(m,n)=\begin{cases}0, & \text{if } (m+n) \text{ is an odd number}\\ -\dfrac{4n}{(m+n+1)(m+n-1)(m-n-1)(m-n+1)}, & \text{if } (m+n) \text{ is an even number}\end{cases}$$

Making use of the formulas (2.7), we write (2.5) in the form

$$F(z)=\frac{h}{2a}\sum_{m=1}^{\infty}\left[\frac{k_0}{k}X_m-iY_m\right]L_m\left(\frac{z}{a}\right)+F_2(z),\quad Q(z)=\frac{h}{2a}\sum_{m=1}^{\circ}\left[\frac{k_0}{k}X_m+iY_m\right]L_m\left(\frac{z}{a}\right)+Q_2(z) \tag{2.9}$$

$$L_m(z)=U_{m-1}(z)-T_m(z)/\sqrt{z^2-1}$$

where ($T_m(z)$ are Chebyshev polynomials of the first kind.

We note that by setting $k_0=0$ in (2.8) and (2.9), we obtain a solution of the problem of heat conduction for a plate with a thermally insulated crack, while putting $k_0=k$ yields a solution of the problem of heat conduction for a plate without an inclusion.

3. **Problem of thermoelasticity.** According to /1/, we can describe the stress-strain state of an isotropic plate by the following formulas:

$$\sigma_x+\sigma_y=2\,[\Phi(z)+\overline{\Phi(z)}],\quad \sigma_y-i\tau_{xy}=\Phi(z)+\Omega(\bar z)+(z-\bar z)\,\overline{\Phi'(z)} \tag{3.1}$$

$$2\mu \frac{\partial}{\partial x}(u + iv) = \varkappa\Phi(z) - \Omega(\bar{z}) - (z - \bar{z})\overline{\Phi'(z)} + \beta\Psi_1(z), \qquad \Psi_1(z) = F_1(z) + Q_1(z)$$

Here $\beta = 2\alpha E$ for the plane deformations and $\beta = 2\alpha E/(1+\nu)$ for the plane state of stress, α is the temperature coefficient of linear expansion, E is the Young's modulus and ν is the Poisson's ratio.

Using the relations (3.1) and conditions (1.1), and following the solution of the problem of heat conduction, we obtain the following boundary value problems for determining the piecewise holomorphic functions $\Phi(z)$ and $\Omega(z)$ with the line of discontinuity L:

$$[\Phi(x) - \Omega(x)]^+ - [\Phi(x) - \Omega(x)]^- = 2ihK'(x), \quad x \in L \tag{3.2}$$

$$[\varkappa\Phi(x) + \Omega(x)]^+ - [\varkappa\Phi(x) + \Omega(x)]^- + \beta(\Psi_1^+(x) - \Psi_1^-(x)) = \frac{2ih\mu}{\mu_0}[M'(x) - M_1(x) + \beta_0\Psi_0'(x)], \quad x \in L$$

$$[\Phi(x) + \Omega(x)]^+ + [\Phi(x) + \Omega(x)]^- = \frac{2}{1+\varkappa_0}[(1-\varkappa_0)K(x) + 2M(x) + 2\overline{K(x)} + 2\overline{M(x)}], \; x \in L \tag{3.3}$$

$$\varkappa[\Phi^+(x) + \Phi^-(x)] - [\Omega^+(x) + \Omega^-(x)] + \beta(\Psi_1^+(x) + \Psi_1^-(x)) =$$
$$\frac{2\mu}{\mu_0(1+\varkappa_0)}[2\varkappa_0 K(x) + (\varkappa_0 - 1)M(x) - 2\overline{K(x)} - 2\overline{M(x)}] + \frac{2\mu\beta_0}{\mu_0}\Psi_0(x), \quad x \in L$$

where

$$M_1(x) = \beta\varepsilon_3\Psi_2'(x), \quad \varepsilon_3 = \frac{\min(\mu_0, \mu)}{\mu}, \qquad \Psi_2(z) = \int F_2(z)\,dz + T_\infty \tag{3.4}$$

$$\Psi_0(x) = \frac{1}{4}\sqrt{a^2 - x^2}\sum_{m=1}^{\infty}\frac{1}{m}(X_m - iY_m)\left[\frac{1}{(m-1)}U_{m-2}\left(\frac{x}{a}\right) - \frac{1}{(m+1)}U_m\left(\frac{x}{a}\right)\right] + \rho_3(x) + T_0$$

$$\Psi_1(z) = \frac{h}{2a}\sum_{m=1}\frac{1}{m}\left(iY_m - \frac{k_0}{k}X_m\right)\sqrt{z^2 - a^2}\,L_m\left(\frac{z}{a}\right) + \Psi_2(z), \qquad \rho_3'(x) = \frac{1}{2}[g_2(x) - i\rho_2(x)]$$

Here T_0 and T_∞ are the temperatures in the inclusion and at infinity, respectively, and $K(x)$, $M(x)$ are unknown functions.

Solving the problem of linear conjugation (3.2), we obtain

$$\Phi(z) = \frac{1}{1+\varkappa}\left\{\frac{h}{\pi}\left[I_K(z) + \frac{\mu}{\mu_0}I_M(z)\right] - \frac{h\beta}{2a}\sum_{m=1}^{\infty}\frac{1}{m}(a_1X_m + ia_2Y_m)\sqrt{z^2 - a^2}\,L_m\left(\frac{z}{a}\right)\right\} \tag{3.5}$$

$$\Omega(z) = \frac{1}{1+\varkappa}\left\{\frac{h}{\pi}\left[-\varkappa I_K(z) + \frac{\mu}{\mu_0}I_M(z)\right] - \frac{h\beta}{2a}\sum_{m=1}^{\infty}\frac{1}{m}(a_1X_m + ia_2Y_m)\sqrt{z^2 - a^2}\,L_m\left(\frac{z}{a}\right)\right\}$$

$$I_K(z) = \int_{-a}^{a}\frac{K'(t)\,dt}{t - z}, \quad I_M(z) = \int_{-a}^{a}\frac{[M'(t) - B(t)]\,dt}{t - z}, \qquad B(t) = M_1(t) - \beta_0\rho_3'(t)$$

$$a_1 = \frac{\mu\beta_0}{\mu_0\beta} - \frac{k_0}{k}, \quad a_2 = 1 - \frac{\mu\beta_0}{\mu_0\beta}$$

Substituting the expressions for the functions $\Phi(z)$ and $\Omega(z)$ (3.5) into the conditions (3.3), we obtain the following system of integrodifferential equations for determining the unknown functions $K(x)$ and $M(x)$:

$$\frac{1}{1+\varkappa_0}[(1-\varkappa_0)K(x) + 2M(x) + 2\overline{K(x)} + 2\overline{M(x)}] - \frac{h(1-\varkappa)}{\pi(1+\varkappa)}I_K(x) - \tag{3.6}$$
$$\frac{2h\mu}{\pi\mu_0(1+\varkappa)}I_M(x) = \frac{h\beta}{(1+\varkappa)}\sum_{m=1}^{\infty}\frac{1}{m}(a_1X_m + ia_2Y_m)T_m\left(\frac{x}{a}\right)$$

$$\frac{\mu}{\mu_0(1+\varkappa_0)}[2\varkappa_0K(x) + (\varkappa_0 - 1)M(x) - 2\overline{K(x)} - 2\overline{M(x)}] - \frac{2h\varkappa}{\pi(1+\varkappa)}I_K(x) - \frac{h\mu(\varkappa-1)}{\pi\mu_0(1+\varkappa)}I_M(x) =$$

$$\frac{h\beta}{2(1+\varkappa)}\sum_{m=1}^{\infty}\frac{1}{m}(b_1X_m - ib_2Y_m)T_m\left(\frac{x}{a}\right) - \frac{\mu\beta_0}{\mu_0}\Psi_0(x) + \beta\Psi_2(x), \quad b_1 = \frac{\mu\beta_0(\varkappa-1)}{\mu_0\beta} + 2\frac{k_0}{k}, \quad b_2 = \frac{\mu\beta_0(\varkappa-1)}{\mu_0\beta} + 2$$

We seek the solution of the system (3.6) in the form

$$K(ax) = K_0 - \sqrt{1-x^2}\sum_{m=1}^{\infty}\frac{1}{m}Z_m U_{m-1}(x), \quad M(ax) = M_0 + B_2(ax) - \sqrt{1-x^2}\sum_{m=1}^{\infty}\frac{1}{m}S_m U_{m-1}(x) \tag{3.7}$$

$$(B_2'(x) = B(x))$$

where K_0, M_0, Z_m, S_m are unknown coefficients.

Performing the necessary manipulations, we arrive at an infinite system of linear algebraic equations for determining the coefficients Z_m and S_m of the expansions

$$\frac{2}{1+\varkappa_0}\sum_{m=1}^{\infty}R(m,n)\left[(1-\varkappa_0)Z_m + 2S_m + 2\overline{Z}_m + 2\overline{S}_m\right] + C_1 Z_n + C_2 S_n = A_n \tag{3.8}$$

$$\frac{2\mu}{\mu_0(1+\varkappa_0)}\sum_{m=1}^{\infty}R(m,n)\left[2\varkappa_0 Z_m + (\varkappa_0-1)S_m - 2\overline{Z}_m - 2\overline{S}_m\right] + C_3 Z_n + C_4 S_n = B_n$$

where

$$H(m,n) = \begin{cases} 0, & n \neq m+1,\ n \neq m-1 \\ \pi/4, & n = m+1 \\ -\pi/4, & n = m-1 \end{cases} \tag{3.9}$$

$$A_n = 2\int_{-1}^{1}\left\{\frac{2}{(1+\varkappa_0)}\left[B_2(ax) + \overline{B_2(ax)}\right] + A_0\right\}\sqrt{1-x^2}\,U_{n-1}(x)\,dx - \frac{2h\beta}{(1+\varkappa)}\sum_{m=1}^{\infty}\frac{1}{m}(a_1X_m + ia_2Y_m)H(m,n)$$

$$B_n = 2\int_{-1}^{1}\left\{\frac{\mu}{\mu_0(1+\varkappa_0)}\left[(\varkappa_0-1)B_2(ax) - 2\overline{B_2(ax)}\right] - \beta\Psi_2(ax) + B_0 + \frac{\mu\beta_0}{\mu_0}(\rho_3(ax) + T_0)\right\}\sqrt{1-x^2}\,U_{n-1}(x)\,dx -$$

$$\frac{h\beta}{(1+\varkappa)}\sum_{m=1}^{\infty}\frac{1}{m}(b_1X_m - ib_2Y_m)H(m,n) + \frac{\mu\beta_0 a}{2\mu_0}\sum_{m=1}^{\infty}\frac{1}{m}(X_m - iY_m)\left[R(m-1,n) - R(m+1,n)\right], \quad n \geqslant 1$$

$$C_1 = \frac{\pi h(1-\varkappa)}{a(1+\varkappa)}, \quad C_2 = \frac{2\pi h\mu}{a\mu_0(1+\varkappa)}, \quad C_3 = \frac{2\pi h\varkappa}{a(1+\varkappa)}, \quad C_4 = \frac{\pi h\mu(\varkappa-1)}{a\mu_0(1+\varkappa)}$$

$$A_0 = \frac{1}{1+\varkappa_0}\left[(1-\varkappa_0)K_0 + 2M_0 + 2\overline{K}_0 + 2\overline{M}_0\right], \quad B_0 = \frac{\mu}{\mu_0(1+\varkappa_0)}\left[2\varkappa_0 K_0 + (\varkappa_0-1)M_0 - 2\overline{K}_0 - 2\overline{M}_0\right]$$

Formulas (3.5) with (3.7) taken into account, yield

$$\Phi(z) = \frac{1}{(1+\varkappa)}\left\{\frac{h}{a}\sum_{m=1}^{\infty}\left(Z_m + \frac{\mu}{\mu_0}S_m\right)L_m\left(\frac{z}{a}\right) - \frac{h\beta}{2a}\sum_{m=1}^{\infty}\frac{1}{m}(a_1X_m + ia_2Y_m)\sqrt{z^2-a^2}\,L_m\left(\frac{z}{a}\right)\right\} \tag{3.10}$$

$$\Omega(z) = \frac{1}{(1+\varkappa)}\left\{\frac{h}{a}\sum_{m=1}^{\infty}\left(-\varkappa Z_m + \frac{\mu}{\mu_0}S_m\right)L_m\left(\frac{z}{a}\right) - \frac{h\beta}{2a}\sum_{m=1}^{\infty}\frac{1}{m}(a_1X_m + ia_2Y_m)\sqrt{z^2-a^2}\,L_m\left(\frac{z}{a}\right)\right\}$$

we assume that the constants A_0 and $\mathrm{Re}B_0$ are $A_0 = \mathrm{Re}B_0 = 0$

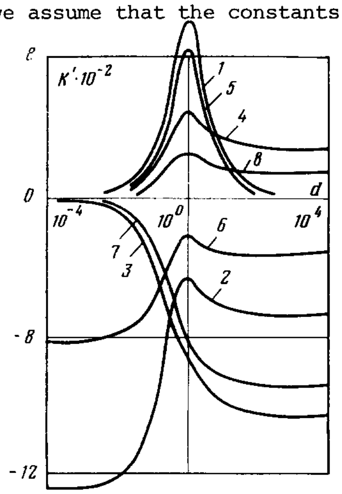

Fig.2

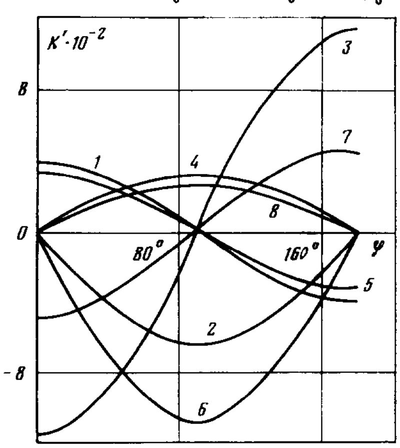

Fig.3

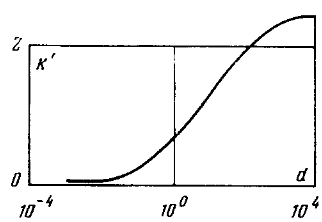

Fig.4

The quantity $\operatorname{Im} B_0$ is found from the condition that the moment of all forces applied to the inclusion is equal to zero (Λ is a closed contour encircling L)

$$\operatorname{Re} \int_{\Lambda} [\overline{\Omega}(z) + \Phi(z)]\, z dz = 0 \qquad (3.11)$$

Using the results of /2,3/, we can show that the system of linear algebraic equations (3.8) is quasiregular.

In analogy with the theory of cracks /4/, we can represent the state of stress in the neighborhood of the end of the inclusion in the following form, using the polar r, θ-coordinate system (Fig.1):

$$\left\| \begin{matrix} \sigma_r \\ \sigma_\theta \\ \tau_{r\theta} \end{matrix} \right\| = \frac{1}{4\sqrt{2r}} [N_1(-1) K_1 + N_2(3) K_2 + N_1(1+2\varkappa) K_3 + N_2(1-2\varkappa) K_4] + O(1) \qquad (3.12)$$

$$N_1(\lambda) = \left\| \begin{matrix} 5\cos {}^1/_2\theta + \lambda \cos {}^3/_2\theta \\ 3\cos {}^1/_2\theta - \lambda\cos {}^3/_2\theta \\ \sin {}^1/_2\theta - \lambda \sin {}^3/_2\theta \end{matrix} \right\|, \quad N_2(\lambda) = \left\| \begin{matrix} -5\sin {}^1/_2\theta + \lambda \sin {}^3/_2\theta \\ -3\sin {}^1/_2\theta - \lambda \sin {}^3/_2\theta \\ \cos {}^1/_2\theta + \lambda\cos {}^3/_2\theta \end{matrix} \right\|$$

Here K_i $(i = 1, 2, 3, 4)$ are the stress intensity coefficients obtained from the formulas ($j = 1$ for the point a, $j = 2$ for the point $-a$)

$$K_1^j - iK_2^j = -\frac{2h\mu}{\mu_0(1+\varkappa)\sqrt{a}} \sum_{m=1}^{\infty} (-1)^{(m+1)(2-j)} S_m, \quad K_3^j - iK_4^j = -\frac{2h}{(1+\varkappa)\sqrt{a}} \sum_{m=1}^{\infty} (-1)^{(m+1)(2-j)} Z_m \qquad (3.13)$$

In conclusion we note, that by assuming in the formulas quoted that $k_0 = k$, $\mu_0 = \mu$, $\varkappa = \varkappa_0$, $\beta = \beta_0$, we obtain a solution of the problem for a plate without an inclusion. On the other hand, passing in the formulas (3.5), (3.6) or (3.8), (3.10) to the limit as $\mu_0 \to 0$ $(k_0 = 0)$ or as $\mu_0 \to \infty$ $(k_0 = 0)$, respectively, where in the second case the condition

$$(\partial v / \partial x - \partial u / \partial y)_0 = 0$$

must also hold, we obtain a solution of the problem of thermoelasticity for a deformable inclusion (a cut) and for a perfectly rigid inclusion. In the particular case when the thermal flux is given at infinity, we obtain the results already given in /5,6/.

4. Results of the numerical analysis. Figs.2—4 depict the results of numerical analysis of the problem for the case of the plate acted upon by a heat flux (q_∞ is the flux intensity at infinity). Computations were carried out for the following values of the parameters: $\nu = \nu_0 = {}^1/_3$, $\alpha_0/\alpha = 0$, $k_0/k = 0$, $a/h = 10$, $T_0 = T_\infty = 0$.

Figure 2 shows the relation between the stress intensity coefficients $K_i' = K_i k/(\beta a^{3/2} q_\infty)$ at the point $x = a$ and relative rigidity of the inclusion $d = \mu_0/\mu$. Curves *1, 3* depict, respectively, K_1' and K_3' for $\varphi = 0$, and curves *2, 4* depict K_2' and K_4' for $\varphi = \pi/2$. In the first case we have $(\varphi = 0)$ $K_2' = K_4' = 0$, and in the second case we have $(\varphi = \pi/2)$ $- K_1' = K_3' = 0$. Curves *5—8* correspond to K_i' $(i = 1, 2, 3, 4)$ for $\varphi = \pi/6$.

Figure 3 depicts the dependence of K_i' $(i = 1, 2, 3, 4)$ on the angle φ at the same point. Curves *1—4* correspond to $K_i'(i = 1, 2, 3, 4)$ for $d = 5$, and curves *5—8* for $d = 0.2$.

Figure 4 shows the dependence of $K_i' = K_i/(\sqrt{a}\beta)$ on the relative rigidity d of the inclusion at $T_0 = T_\infty = 5$ (the corresponding curve for $T_0 = T_\infty = -5$ is symmetrical to the previous curve about the abscissa). In this case we have $K_2' = K_4' = 0$, $K_1' \approx 0$. The computations were carried out for $q_\infty = 0$, $\nu = \nu_0 = {}^1/_3$, $\alpha_0/\alpha = 0$, $a/h = 10$.

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